International Journal of Theoretical Physics, Vol. 39, No. 10, 2000

Photon–Dyon Scattering in Non-Abelian Gauge Theory

P. P. Purohit,[1] V. P. Pandey,[1] and B. S. Rajput[1]

Received July 5, 2000

We study the scattering of a non-Abelian dyon and photons. We demonstrate that two photons are necessary for Compton scattering of a non-Abelian dyon, through *S*-matrix expansion. One of these two photons is associated with electric four-potential and is ordinary, while the other is associated with magnetic four-potential and is highly energetic and responsible for scattering of magnetic charge of the non-Abelian dyon. We also study dyon–photon, dyon–dyon, and dyon–antidyon scattering and the self-energy of the dyon and both photons in non-Abelian gauge theories.

1. INTRODUCTION

The renewed interest in the theory of the monopole and the dyon is partly due to the work of 't Hooft[(1)] and Polyakov,[(2)] who embedded the $U(1)$ electromagnetic field in $SU(2)$ gauge theory and obtained numerical solutions of the canonical finite-mass monopole in the whole space through spontaneous symmetry breaking due to the Higgs field, which leaves behind unbroken $U(1)$ gauge symmetry. Julia and Zee[(3)] obtained the corresponding numerical solutions for a dyon, and Prasad and Sommerfield[(4)] derived the analytic solutions for the monopoles and dyons of finite mass by keeping the symmetry of the vacuum broken but letting the self-interaction of the Higgs field approach zero. It is widely recognized[(5)] that the $SU(5)$ grand unified model[(6)] is a gauge theory that contains magnetic monopole solutions, and consequently the question of the existence of the monopole and the dyon has gathered enormous potential importance in connection with the problem of quark confinement,[(7)] possible magnetic condensation of the vacuum,[(8)] their role in catalyzing proton decay,[(9,10)] and the possible explanation of *CP* violation.[(11)] Keeping in view

[1] Department of Physics, Kumaun University, Nainital 263 002, India.

0020-7748/00/1000-2457$18.00/0 © 2000 Plenum Publishing Corporation

all these facts and the observation of Cabrera,(12) we have constructed(13,14) a manifestly covariant quantum field theory of dyons each carrying generalized charge as a complex quantity with electric and magnetic charges as its real and imaginary parts, formulated a non-Abelian theory of dyons,(15,16) and investigated bound states of two dyons and dyon–fermion(17,18) in nonrelativistic as well as in relativistic frameworks. We have also undertaken the study of the scattering of photons by the monopole and the dyon in Abelian theory and have shown that two photons are necessary for Compton scattering of dyons through *S*-matrix expansion and that photons associated with the monopole and the dyon have enormously high energy.(19–21) Study of the bound state of dyon and antidyon has also been carried out and it has been shown that this state is very short lived and decays into four or six photons, depending on the spin statistics of the dyons involved.(19,20) Extending this work in the present paper, we study dyon–photon scattering in non-Abelian gauge theory with the help of *S*-matrix expansion for this system.

2. COMPTON SCATTERING OF PHOTONS AND A NON-ABELIAN DYON

In order to study the Compton scattering of photons and a non-Abelian dyon we assume that the non-Abelian dyon is at rest and there are photons incident on it. It is reasonable to expect that low-energy photons will make very little impact on the non-Abelian dyon, which is very heavy. Further, as shown in the next section [Eq. (3.34)], we need two photons to make Compton scatterring possible for this system. These two photons differ from each other essentially, as one is associated with electric four-potential and the other is associated with magnetic four-potential. This scattering process is shown in Fig. 1. Applying the laws of conservation of energy and momentum, we get

$$(1 + k)h\nu = (1 + k)h\nu' + M_{\mathrm{Do}}c^2\left[\frac{1}{(1 - v^2/c^2)^{1/2}} - 1\right] \tag{2.1}$$

and

$$\frac{(1 + k)h\nu}{c} = \frac{(1 + k)h\nu'}{c}\cos\phi + \frac{M_{\mathrm{Do}}v}{(1 - v^2/c^2)^{1/2}}\cos\theta \qquad \text{along } x \text{ axis} \tag{2.2}$$

$$\frac{(1 + k)h\nu'}{c}\sin\phi - \frac{M_{\mathrm{Do}}v}{(1 - v^2/c^2)^{1/2}}\sin\theta = 0 \qquad \text{along } y \text{ axis} \tag{2.3}$$

where $k = M_{\mathrm{Do}}/M$; M_{Do} is the rest mass of a non-Abelian dyon and M is the rest mass of the electron. With the help of these equations, we get the change in wavelength $\Delta\lambda = \lambda' - \lambda$ as

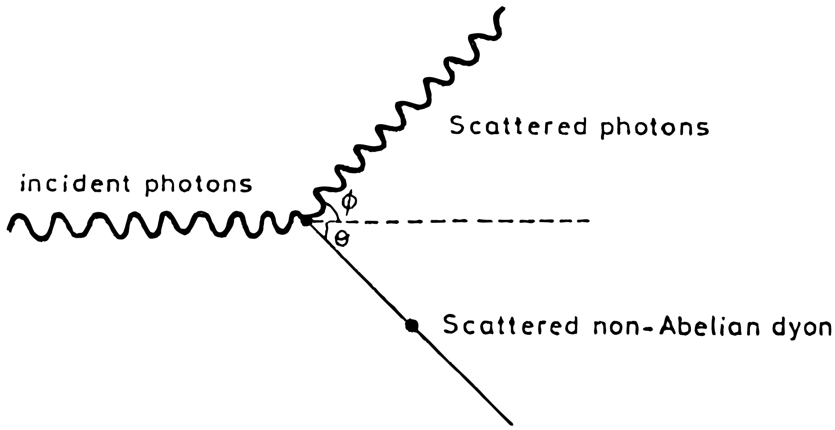

Fig. 1. Compton scattering. Black lines denote ordinary photon and red lines denote highly energetic photon. [This convention of denoting different photons will be used throughout the figures.]

$$\Delta\lambda = \frac{(1 + k)h}{M_{\mathrm{Do}}c}(1 - \cos\phi) \tag{2.4}$$

which is the Compton shift for a non-Abelian dyon. The direction of the scattered non-Abelian dyon and the energy are, respectively,

$$\tan\theta = \frac{1}{1 + \alpha}\cot\left(\frac{\phi}{2}\right) \tag{2.5}$$

and

$$E_k = (1 + k)h\nu\left[\frac{2\alpha\sin^2(\phi/2)}{1 + 2\alpha\sin^2(\phi/2)}\right] \tag{2.6}$$

where

$$\alpha = \frac{(1 + k)h\upsilon}{M_{\mathrm{Do}}c^2}$$

These results show that one of these two photons is ordinary and is responsible for $\gamma + e \rightarrow \gamma + e$ scattering, its energy is $h\upsilon$ (1.989×10^{-15} J). The second photon is responsible for $\gamma + g \rightarrow \gamma + g$ scattering and its energy is $kh\upsilon$ (2.42906×10^{14} GeV). This enormously high energy is due to the heavy mass of the magnetic monopole, which is of the order $\sim 10^{16}$ GeV.[22]

3. *S*-MATRIX EXPANSION

In this section we study dyon–photon scattering in non-Abelian gauge theory with the help of the S-matrix expansion technique, so that the generalization of Abelian gauge theory[19–21] can be investigated in the three-dimensional representation of isotopic space, where the complex charge space is locked together with the internal isotopic space in such a way that the charge, electric as well as magnetic, spreads over the components of internal space. The generators of gauge transformations in internal charge space may be considered as T^a, with $a = 1, 2, 3$, for $SO(3)$ internal symmetry:

$$(T^c)_{ab} = -i\varepsilon_{abc} \tag{3.1}$$

where $\boldsymbol{\epsilon}_{abc}$ is the usual Levi-Civita symbol. These matrices satisfy the following commutation rule:

$$[T^a, T^b] = i\varepsilon_{abc}T^c \tag{3.2}$$

The charge operator in the internal space may be defined as $\hat{r} \cdot \vec{T}$, which in case of a complex charge takes the form $\hat{r} \cdot (\vec{T}^1 - i\vec{T}^2)$ as a result of locking between internal space and complex charge space. Through a specific kind of gauge transformation which sends the operator $\hat{r} \cdot \vec{T}$ to an operator $\vec{T}^3$, the interlocking of external and internal space may be removed. The position-dependent gauge transformation is singular, but does not introduce any singularity in the field. The charge multiplet thus interacts with the multiplets of the generalized Yang–Mills field. In the relativistic framework, the complex isospin space further splits up into two-dimensional spin space. It reveals new characteristics of scattering which are not present in the theory of spin-0 dyons. The Lagrangian density for such a system is

$$L = L_0 + i\overline{\Psi}\hat{\gamma}_\mu D_\mu \Psi - Gq^+\overline{\Psi}T^a\phi^a\Psi \tag{3.3}$$

where L_0 is the 't Hooft–Polyakov Lagrangian[1,2] and

$$iD_\mu = i\partial_\mu - q^* V_\mu^a T^a \tag{3.4}$$

V_μ^a in Eq. (3.4) is the matrix form of generalized four-potential of the non-Abelian dyon and is given by

$$V_\mu^a = A_\mu^a - iB_\mu^a \tag{3.5}$$

where A_μ^a and B_μ^a are the electric and magnetic four-potential and T^a are matrices given by (3.1). The third term in Eq. (3.3) includes the mass of the dyon in terms of the Higgs scalar field, and G is the coupling constant between the Higgs triplet and isospin triplet. With minimal replacement of Eq. (3.4), the Lagrangian density takes the following form:

$$\hat{L}(x) = \hat{L}_0(x) + i\hat{\bar{\Psi}}(x)\hat{\gamma}_\mu(\partial_\mu + iq^* V_\mu^a T^a)\hat{\Psi}(x) - Gq^*\hat{\bar{\Psi}}(x)T^a\hat{\gamma}^a(x)\hat{\Psi}(x) \tag{3.6}$$

which includes the interaction Lagrangian density

$$\hat{L}_I(x) = -\hat{\bar{\Psi}}(x)\hat{\gamma}_\mu q^* V_\mu^a T^a \hat{\Psi}(x) \tag{3.7}$$

where

$$q^* V_\mu^a = (eA_\mu^a + gB_\mu^a) + i(gA_\mu^a - eB_\mu^a) \tag{3.8}$$

and

$$q = e - ig \tag{3.9}$$

is the generalized charge of the dyon. Finally, we get the following interaction Lagrangian density by substituting the real part of Eq. (3.8) into Eq. (3.7)

$$\hat{L}_I(x) = -e\hat{\bar{\Psi}}_e(x)\hat{\gamma}_\mu \hat{A}_\mu^a(x)T^a\hat{\Psi}_e(x) - g\hat{\bar{\Psi}}_g(x)\hat{\gamma}_\mu \hat{B}_\mu^a(x)T^a\hat{\Psi}_g(x) \tag{3.10}$$

The interaction Hamiltonian density may be deduced in the form

$$\begin{aligned}\hat{H}_I(x) &= e\hat{\bar{\Psi}}_e(x)\hat{\gamma}_\mu \hat{A}_\mu^a(x)T^a\hat{\Psi}_e(x) + g\hat{\bar{\Psi}}_g(x)\hat{\gamma}_\mu \hat{B}_\mu^a(x)T^a\hat{\Psi}_g(x)\\ &= eN(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\hat{\Psi}_e)_x + gN(\hat{\bar{\Psi}}_g\hat{\gamma}_\mu\hat{B}_\mu^a T^a\hat{\Psi}_g)_x\end{aligned} \tag{3.11}$$

In order to find perturbative series solutions, we write the *S*-matrix expansion by choosing the perturbation Hamiltonian $\hat{H}_I(t)$ in the interaction picture as

$$\hat{S} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \ldots \int_{-\infty}^{\infty} dt_n\, P[\hat{H}_I(t_1)\hat{H}_I(t_2)\ldots\hat{H}_I(t_n)] \tag{3.12}$$

where P is the Dyson chronological product and $\hat{H}_I(t)$ is defined as

$$\hat{H}_I(t) = \int dx\, \hat{H}_I(x) \tag{3.13}$$

$\hat{H}_I(x)$ of this equation is given by Eq. (3.11) upon substituting the value of $H_I(t)$ from Eq. (3.13) into Eq. (3.12); we get

$$\hat{S} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n\, T[\hat{H}_I(x_1)\hat{H}_I(x_2)\ldots\hat{H}_I(x_n)] \tag{3.14}$$

where T denotes Wick's chronological product. With the help of Eqs. (3.11) and (3.14), we get

$$\begin{aligned}\hat{S} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} e^n \int d^4 x_1 \int d^4x_2 \ldots \int d^4x_n\, T'[(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\hat{\Psi}_e)_{x_1}\\ \times (\hat{\bar{\Psi}}_e\hat{\gamma}_\nu\hat{A}_\nu^b T^b\hat{\Psi}_e)_{x_2} \ldots (\hat{\bar{\Psi}}_e\hat{\gamma}_\rho\hat{A}_\rho^c T^c\hat{\Psi}_e)_{x_n}]\end{aligned}$$

$$+ \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} g^n \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n \, T'[(\hat{\overline{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} \times (\hat{\overline{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2} \ldots (\hat{\overline{\Psi}}_g \hat{\gamma}_\rho \hat{B}^c_\rho T^c \hat{\Psi}_g)_{x_n}] \tag{3.15}$$

We can now discuss various processes arising from the terms of various orders in the *S*-matrix expansion (3.15) for the interaction of a non-Abelian dyon with the generalized electromagnetic field of another non-Abelian dyon. When $n = 0$, we have

$$\hat{S}^{(0)} = 1 + 1 = 2 \tag{3.16}$$

for which no scattering takes place and hence the Feynman diagram contains no vertex. For $n = 1$, we have

$$\hat{S}^{(1)} = -ie \int d^4x \, T'(\hat{\overline{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} - ig \int d^4x \, T'(\hat{\overline{\Psi}}_e \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} \tag{3.17}$$

where

$$\hat{\overline{\Psi}} = \hat{\overline{\Psi}}^- + \hat{\overline{\Psi}}^+ \; ; \qquad \hat{\Psi} = \hat{\Psi}^+ + \hat{\Psi}^-$$
$$\hat{A}^a_\mu = \hat{A}^{a+}_\mu + \hat{A}^{a-}_\mu ; \qquad \hat{B}^a_\mu = \hat{B}^{a+}_\mu + \hat{B}^{a-}_\mu$$

with $\hat{\overline{\Psi}}^-$ and $\hat{\overline{\Psi}}^+$ respectively denoting non-Abelian dyon creation and non-Abelian antidyon annihilation and $\hat{\overline{\Psi}}^+$ and $\hat{\overline{\Psi}}^-$ denoting non-Abelian dyon annihilation and non-Abelian antidyon creation, respectively. We further split $\hat{\overline{\Psi}}^-$ and $\hat{\overline{\Psi}}^+$ by denoting $\hat{\overline{\Psi}}^-_e$ as electron creation and $\hat{\overline{\Psi}}^-_g$ as monopole creation and $\hat{\overline{\Psi}}^+_e$ as positron annihilation and $\hat{\overline{\Psi}}^+_g$ as antimonopole annihilation in non-Albelian gauge theory. Similarly we can defined $\hat{\Psi}^+$ and $\hat{\Psi}^-$. The explicit expansions for $\hat{\Psi}_e(x)$ and $\hat{\Psi}_g(x)$ are given by

$$\hat{\Psi}_e(x) = \begin{pmatrix} \hat{\Psi}^{1/2}_{1,1/2} \\ \hat{\Psi}^{1/2}_{2,-1/2} \end{pmatrix}_e = \frac{1}{\sqrt{V}} \sum_p \sqrt{\frac{m_e}{E_p}} \sum_{r=1}^{2} [\hat{c}_{re}(\vec{p}) u_{re}(\vec{p}) e^{ipx} + \hat{d}^+_{re}(\vec{p}) v_{re}^{(\vec{p})} \, e^{-ipx}]_e = \hat{\Psi}^+_e(x) + \hat{\Psi}^-_e(x) \tag{3.18}$$

where

$$(\hat{\Psi}^{1/2}_{1,1/2})_e = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_e = U_{1e}; \qquad (\hat{\Psi}^{1/2}_{2,-1/2})_e = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_e = U_{2e} \tag{3.19}$$

and

$$\hat{\Psi}_g(x) = \begin{pmatrix} \hat{\Psi}^{1/2}_{1,1/2} \\ \hat{\Psi}^{1/2}_{2,-1/2} \end{pmatrix}_g = \frac{1}{\sqrt{V}} \sum_p \sqrt{\frac{m_g}{E_p}} \sum_{r=1}^{2} [\hat{c}_{rg}(\vec{p}) u_{rg}(\vec{p}) e^{ipx}$$
$$+ \hat{d}^{+}_{rg}(\vec{p}) v^{(\vec{p})}_{rg}\, e^{-ipx}]_g$$
$$= \hat{\Psi}^{+}_g(x) + \hat{\Psi}^{-}_g(x) \tag{3.20}$$

with

$$(\hat{\Psi}^{1/2}_{1,1/2})_g = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_g = U_{1g}; \qquad (\hat{\Psi}^{1/2}_{2,-1/2})_g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_g = U_{2g} \tag{3.21}$$

where m_e and m_g are the masses of electric and magnetic charges of the dyon in non-Abelian gauge theory, and

$$\hat{\bar{\Psi}}_e(x) = \frac{1}{\sqrt{V}} \sum_p \sqrt{\frac{m_e}{E_p}} \sum_{r=1}^{2} [\hat{c}^{+}_{re}(\vec{p}) \bar{u}_{re}(\vec{p}) e^{-ipx} + \hat{d}_{re}(\vec{p}) \bar{v}_{re}(\vec{p}) e^{ipx}]_e$$
$$= \hat{\bar{\Psi}}^{-}_e(x) + \hat{\bar{\Psi}}^{+}_e(x) \tag{3.22}$$

$$\hat{\bar{\Psi}}_g(x) = \frac{1}{\sqrt{V}} \sum_p \sqrt{\frac{m_g}{E_p}} \sum_{r=1}^{2} [\hat{c}^{\dagger}_{rg}(\vec{p}) \bar{u}_{rg}(\vec{p}) e^{-ipx} + \hat{d}_{rg}(\vec{p}) \bar{v}_{rg}(\vec{p}) e^{ipx}]_g$$
$$= \hat{\bar{\Psi}}^{-}_g(x) + \hat{\bar{\Psi}}^{+}_g(x) \tag{3.23}$$

Similarly, $\hat{A}^{a+}_\mu(x)$, $\hat{A}^{a-}_\mu(x)$, $\hat{B}^{a+}_\mu(x)$, and $\hat{B}^{a-}_\mu(x)$ can be defined in the following way:

$$\hat{A}^{a+}_\mu(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}_e} \frac{1}{\sqrt{2\omega_{\vec{k}_e}}} \sum_{r=1}^{4} [\hat{a}^a_r(\vec{k}_e) \varepsilon^{(r)}_{\mu e}(\vec{k}_e) e^{ik_e x}] \tag{3.24}$$

$$\hat{A}^{a-}_\mu(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}_e} \frac{1}{\sqrt{2\omega_{\vec{k}_e}}} \sum_{r=1}^{4} [\hat{a}^{a+}_r(\vec{k}_e) \varepsilon^{(r)*}_{\mu e}(\vec{k}_e) e^{-ik_e x}] \tag{3.25}$$

$$\hat{B}^{a+}_\mu(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}_g} \frac{1}{\sqrt{2\omega_{\vec{k}_g}}} \sum_{r=1}^{4} [\hat{b}^a_r(\vec{k}_g) \varepsilon^{(r)}_{\mu g}(\vec{k}_g) e^{ik_g x}] \tag{3.26}$$

$$\hat{B}^{a-}_\mu(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}_g} \frac{1}{\sqrt{2\omega_{\vec{k}_g}}} \sum_{r=1}^{4} [\hat{b}^{a\dagger}_r(\vec{k}_g) \varepsilon^{(r)*}_{\mu g}(\vec{k}_g) e^{-ik_g x}] \tag{3.27}$$

where $\varepsilon^{(r)}_\mu$ is the μ^{th} component of $\varepsilon^{(r)}(\vec{k})$ (i.e., projection of $\varepsilon^{(r)}$ on the x_μ

axis) in Minkowski space. The subscript e or g denotes whether it is related to the electric charge or magnetic charge of the non-Abelian dyon, respectively. We can interpret $\hat{a}_r^a(\vec{k}_e)$, $\hat{a}_r^{a\dagger}(\vec{k}_e)$ and $\hat{a}_r^{a\dagger}(\vec{k}_e)\hat{a}_r^a(\vec{k}_e)$ as annihilation, creation, and number of operators for the electric part of the non-Abalian-dyon and $\hat{b}_r^a(\vec{k}_g)$, $\hat{b}_r^{a\dagger}(\vec{k}_g)$, and $\hat{b}_r^{a\dagger}(\vec{k}_g)\hat{b}_r^a(\vec{k}_g)$ as the annihilation, creation, and number operators for the magnetic counterpart of non-Abelian dyon. The photons corresponding to electric charge have momentum $\vec{k}_e$, energy $\omega_{\vec{k}_e} = |\vec{k}_e|$ and polarization vector $\varepsilon_e^{(r)}(\vec{k}_e)$, whereas photons corresponding to magnetic charge have momentum $\vec{k}_g$, energy $\omega_{\vec{k}_g} = |\vec{k}_g|$, and polarization vector $\varepsilon_g^{(r)}(\vec{k}_g)$.

The first-order S-matrix $\hat{S}^{(1)}$ given by Eq. (3.17) gives rise to eight basic processes (Feynman diagrams) which do not lead to any physical process, as they would violate the conservation laws. We write an S-matrix process out of eight basic processes in the following way:

$$T'(\hat{\bar{\Psi}}_e^-\hat{\gamma}_\mu\hat{A}_\mu^{a+}T^a\hat{\Psi}_e^+)_{x_1} + T'(\hat{\bar{\Psi}}_g^-\hat{\gamma}_\mu\hat{B}_\mu^{a+}T^a\hat{\Psi}_g^+)_{x_1} \tag{3.28}$$

This is shown in Fig. 2 which describes dyon scattering by the absorption of photons in non-Abelian gauge theory. For describing real processes, let us take the second-order S-matrix $\hat{S}^{(2)}$ in Eq. (3.15) and then we have for $n = 2$:

$$\begin{aligned}\hat{S}^{(2)} = & -\frac{e^2}{2!}\int d^4x_1 \int d^4x_2\ T'[(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\hat{\Psi}_e)_{x_1}(\hat{\bar{\Psi}}_e\hat{\gamma}_\nu\hat{A}_\nu^b T^b\hat{\Psi}_e)_{x_2}] \\ & -\frac{g^2}{2!}\int d^4x_1 \int d^4x_2\ T'[(\hat{\bar{\Psi}}_g\hat{\gamma}_\mu\hat{B}_\mu^a T^a\hat{\Psi}_g)_{x_1}(\hat{\bar{\Psi}}_g\hat{\gamma}_\nu\hat{B}_\nu^b T^b\hat{\Psi}_g)_{x_2}]\end{aligned} \tag{3.29}$$

By the application of Wick's theorem, we get

$$\hat{S}^{(2)} = \sum_{i=1}^{6}\hat{S}_i^{(2)} \tag{3.30}$$

where

$$\begin{aligned}\hat{S}_1^{(2)} = & -\frac{e^2}{2!}\int d^4x_1 \int d^4x_2\ N[(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\hat{\Psi}_e)_{x_1}(\hat{\bar{\Psi}}_e\hat{\gamma}_\nu\hat{A}_\nu^b T^b\hat{\Psi}_e)_{x_2}] \\ & -\frac{g^2}{2!}\int d^4x_1 \int d^4x_2\ N[(\hat{\bar{\Psi}}_g\hat{\gamma}_\mu\hat{B}_\mu^a T^a\hat{\Psi}_g)_{x_1}(\hat{\bar{\Psi}}_g\hat{\gamma}_\nu\hat{B}_\nu^b T^b\hat{\Psi}_g)_{x_2}]\end{aligned} \tag{3.31a}$$

$$\begin{aligned}\hat{S}_2^{(2)} = & -\frac{e^2}{2!}\int d^4x_1 \int d^4x_2\ \{N[(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\underbrace{\hat{\Psi}_e)_{x_1}(\hat{\bar{\Psi}}}_e\hat{\gamma}_\nu\hat{A}_\nu^b T^b\hat{\Psi}_e)_{x_2}] \\ & + N[(\underbrace{\hat{\bar{\Psi}}_e\hat{\gamma}_\mu\hat{A}_\mu^a T^a\hat{\Psi}_e)_{x_1}(\hat{\bar{\Psi}}_e\hat{\gamma}_\nu\hat{A}_\nu^b T^b\hat{\Psi}}_e)_{x_2}]\}\end{aligned}$$

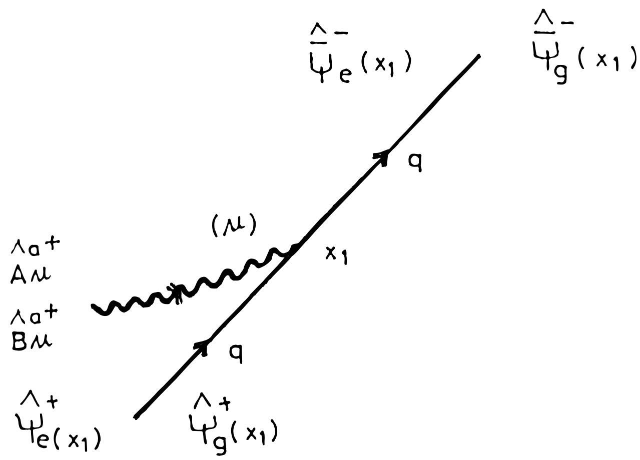

Fig. 2. Dyon scattering by photon absorption in non-Abelian gauge theory.

$$
\begin{aligned}
&- \frac{g^2}{2!} \int d^4x_1 \int d^4x_2 \, \{N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}] \\
&+ N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}]\}
\end{aligned} \tag{3.31b}
$$

$$
\begin{aligned}
\hat{S}^{(2)}_3 = &-\frac{e^2}{2!} \int d^4x_1 \int d^4x_2 \, N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}] \\
&- \frac{g^2}{2!} \int d^4x_1 \int d^4x_2 \, N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}]
\end{aligned} \tag{3.31c}
$$

$$
\begin{aligned}
\hat{S}^{(2)}_4 = &-\frac{e^2}{2!} \int d^4x_1 \int d^4x_2 \, \{N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}] \\
&+ N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}]\} \\
&- \frac{g^2}{2!} \int d^4x_1 \int d^4x_2 \, \{N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}] \\
&+ N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}]\}
\end{aligned} \tag{3.31d}
$$

$$
\hat{S}^{(2)}_5 = -\frac{e^2}{2!} \int d^4x_1 \int d^4x_2 \, N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}]
$$

$$- \frac{g^2}{2!} \int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}] \tag{3.31e}$$

$$\hat{S}^{(2)}_6 = -\frac{e^2}{2!} \int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}]$$

$$- \frac{g^2}{2!} \int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}] \tag{3.31f}$$

The first of these terms, $\hat{S}^{(2)}_1$, given by Eq. (3.31a) does not contain a propagator between the vertices x_1 and x_2 and hence contains two unconnected basic vertex parts. As such it does not lead to any real process.

We can write $\hat{S}^{(2)}_2$ as

$$\hat{S}^{(2)}_2 = -e^2 \int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_e \hat{\gamma}_\mu \hat{A}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\hat{\bar{\Psi}}_e \hat{\gamma}_\nu \hat{A}^b_\nu T^b \hat{\Psi}_e)_{x_2}]$$

$$- g^2 \int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g \hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\hat{\bar{\Psi}}_g \hat{\gamma}_\nu \hat{B}^b_\nu T^b \hat{\Psi}_g)_{x_2}] \tag{3.32}$$

In order to discuss the various processes involved in this expression, we write it as follows:

$$\hat{S}^{(2)}_2 = \sum_{i=a}^{f} \hat{S}^{(2)}_{2i} \tag{3.33}$$

where

$$\hat{S}^{(2)}_{2a} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}^-_e(x_1) \hat{\gamma}_\mu T^a S^{ab}_F(x_1 - x_2) \hat{\gamma}_\nu T^b A^{a-}_\mu(x_1) \hat{A}^{b+}_\nu(x_2) \hat{\Psi}^+_e(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}^-_g(x_1) \hat{\gamma}_\mu T^a S^{ab}_F(x_1 - x_2) \hat{\gamma}_b T^b \hat{B}^{a-}_\mu(x_1) \hat{B}^{b+}_\nu(x_2) \hat{\Psi}^+_g(x_2) \tag{3.33a}$$

$$\hat{S}^{(2)}_{2b} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}^-_e(x_1) \hat{\gamma}_\mu T^a S^{ab}_F(x_1 - x_2) \hat{\gamma}_\nu T^b \hat{A}^{b-}_\nu(x_2) \hat{A}^{a+}_\mu(x_1) \hat{\Psi}^+_e(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}^-_g(x_1) \hat{\gamma}_\mu T^a S^{ab}_F(x_1 - x_2) \hat{\gamma}_\nu T^b \hat{B}^{b-}_\nu(x_2) \hat{B}^{b+}_\mu(x_1) \hat{\Psi}^+_g(x_2) \tag{3.33b}$$

$$\hat{S}^{(2)}_{2c} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{\Psi}^-_e(x_1) \hat{\gamma}_\mu T^a S^{ab}_F(x_1 - x_2) \hat{\gamma}_\nu T^b \hat{A}^{a-}_\mu(x_1) \hat{A}^{b+}_\nu(x_2) \hat{\bar{\Psi}}^+_e(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{\Psi}_g^-(x_1)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{B}_\mu^{a-}(x_1)\hat{B}_\nu^{b+}(x_2)\hat{\bar{\Psi}}_g^+(x_2) \tag{3.33c}$$

$$\hat{S}_{2d}^{(2)} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{\Psi}_e^-(x_1)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{A}_\nu^{b-}(x_2)\hat{A}_\mu^{a+}(x_1)\hat{\bar{\Psi}}_e^+(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{\Psi}_g^-(x_1)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{B}_\nu^{b-}(x_2)\hat{B}_\mu^{a+}(x_1)\hat{\bar{\Psi}}_g^+(x_2) \tag{3.33d}$$

$$\hat{S}_{2e}^{(2)} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}_e^-(x_1)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{\Psi}_e^-(x_2)\hat{A}_\mu^{a+}(x_1)\hat{A}_\nu^{b+}(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{\bar{\Psi}}_g^-(x_1)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{\Psi}_g^-(x_2)\hat{B}_\mu^{a+}(x_1)\hat{B}_\nu^{b+}(x_2) \tag{3.33e}$$

$$\hat{S}_{2f}^{(2)} = ie^2 \int d^4x_1 \int d^4x_2\, \hat{A}_\mu^{a-}(x_1)\hat{A}_\mu^{b-}(x_2)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{\bar{\Psi}}_e^+(x_1)\hat{\Psi}_e^+(x_2)$$

$$+ ig^2 \int d^4x_1 \int d^4x_2\, \hat{B}_\mu^{a-}(x_1)\hat{B}_\nu^{b-}(x_2)\hat{\gamma}_\mu T^a S_F^{ab}(x_1 - x_2)\hat{\gamma}_\nu T^b \hat{\bar{\Psi}}_g^+(x_1)\hat{\Psi}_g^+(x_2) \tag{3.33f}$$

Equations (3.33a) and (3.33b) correspond to Compton scattering by dyons in non-Abelian gauge theory

$$\gamma + \gamma + e + g \rightarrow \gamma + \gamma + e + g \tag{3.34}$$

The corresponding diagram is shown in Figs. 3a and 3b. Equations (3.33c) and (3.33d) correspond to Compton scattering by the antidyon in non-Abelian gauge theory (as shown in Figs. 3c and 3d),

$$\gamma + \gamma + e^+ + \bar{g} \rightarrow \gamma + \gamma + e^+ + \bar{g} \tag{3.35}$$

and Eq. (3.33e) leads to the non-Abelian dyon and non-Abelian antidyon pair creation process (as shown in Fig. 4a),

$$\gamma + \gamma + \gamma + \gamma \rightarrow e + e^+ + g + \bar{g} \tag{3.36}$$

Similarly, Eq. (3.33f) describes non-Abelian dyon and non-Abelian anti-dyon annihilation (as shown in Fig. 4b),

$$e + e^+ + g + \bar{g} \rightarrow \gamma + \gamma + \gamma + \gamma \tag{3.37}$$

The term $\hat{S}_3^{(2)}$ gives rise to the following scattering processes:

a

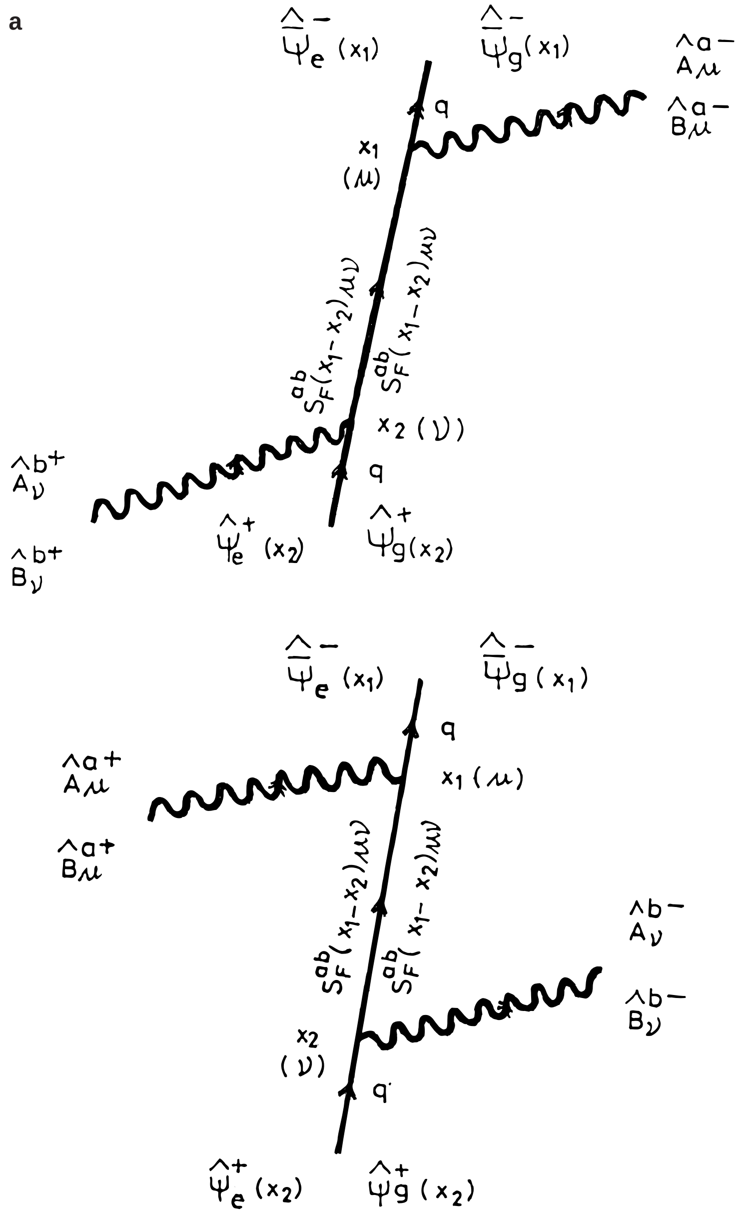

Fig. 3. Compton scattering by a dyon and an antidyon in non-Abelian gauge theory. Here $\hat{A}_{\nu}^{b+}$ and $\hat{A}_{\mu}^{a-}$ represent the absorption and emission of the photon corresponding to the electric charge on a non-Abelian dyon and $\hat{B}_{\nu}^{b+}$ and $\hat{B}_{\mu}^{b-}$ represent the absorpton and emission of the photon corresponding to the magnetic charge on a non-Abelian dyon.

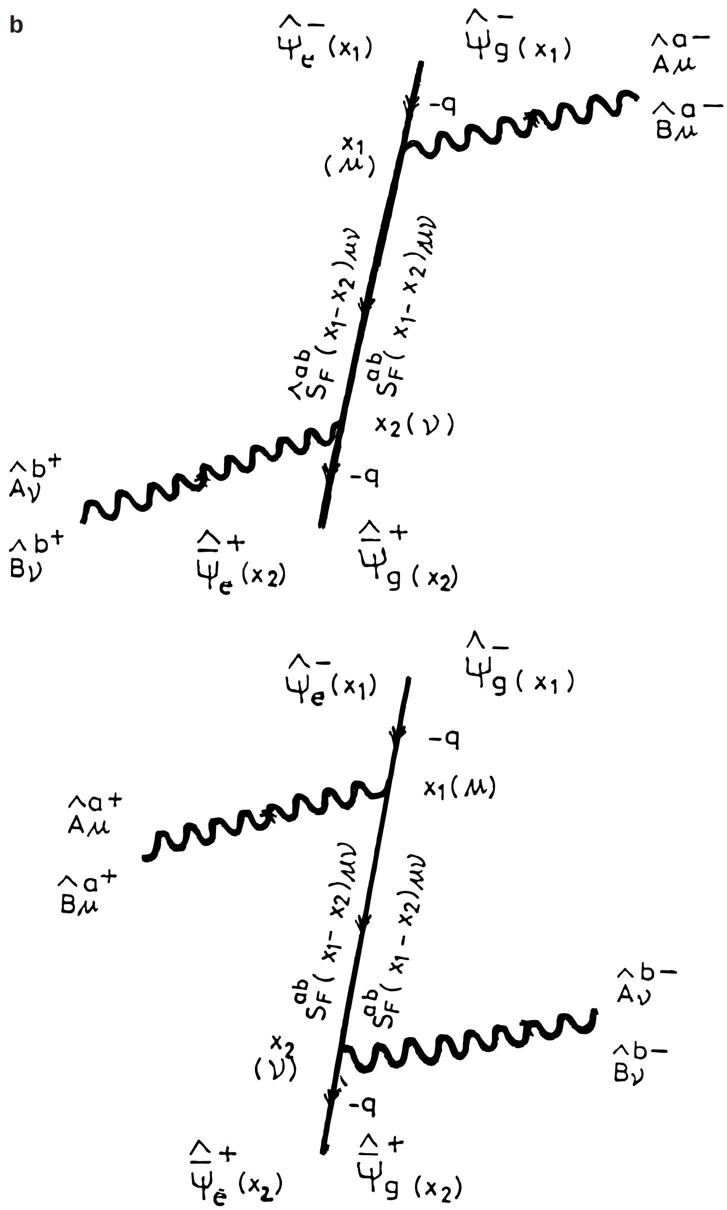

Fig. 3. Continued.

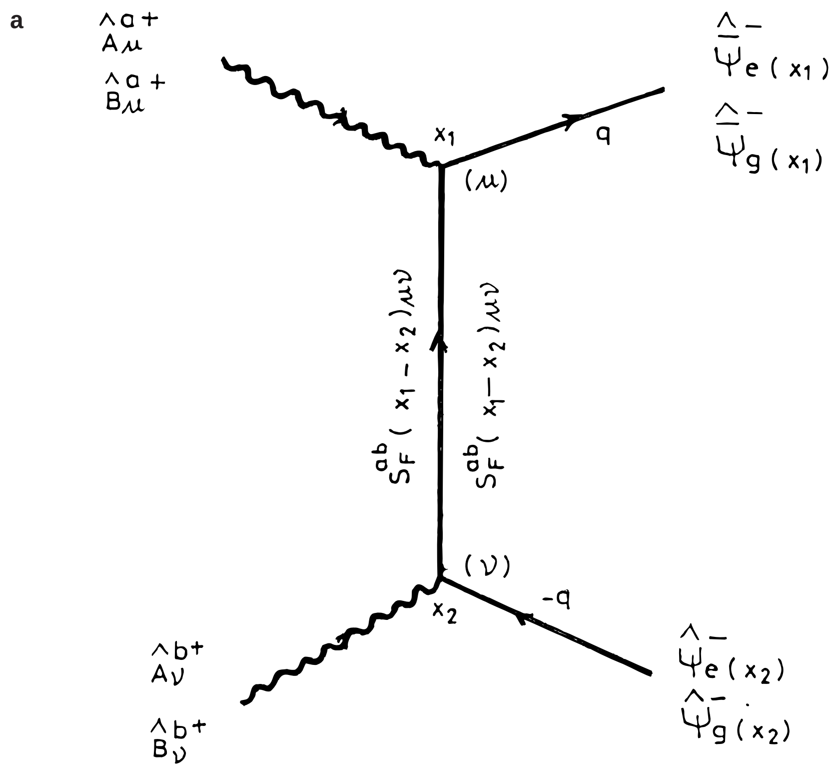

Fig. 4. Non-Abelian dyon–non Abelian antidyon (a) pair creation and (b) pair annihilation.

A. Dyon–dyon scattering in non-Abelian gauge theory:

$$e + e + g + g \rightarrow e + e + g + g \tag{3.38}$$

We can describe this process by the equation

$$\begin{aligned}\hat{S}^{(2)}_{3D} = &-\frac{ie^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^-_e \hat{\gamma}_\mu T^a \hat{\Psi}^+_e)_{x_1}(\hat{\bar{\Psi}}^-_e \hat{\gamma}_\nu T^b \hat{\Psi}^+_e)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\nu} \\ &- \frac{ig^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^-_g \hat{\gamma}_\mu T^a \hat{\Psi}^+_g)_{x_1}(\hat{\bar{\Psi}}^-_g \hat{\gamma}_\nu T^b \hat{\Psi}^+_g)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\nu}\end{aligned} \tag{3.39}$$

We show this process in Fig. 5a.

B. Dyon–antidyon scattering in non-Abelian gauge theory:

$$e + e^+ + g + \bar{g} \rightarrow e + e^+ + g + \bar{g} \tag{3.40}$$

The part of $\hat{S}^{(2)}_3$ which contributes to this process contains eight uncontracted

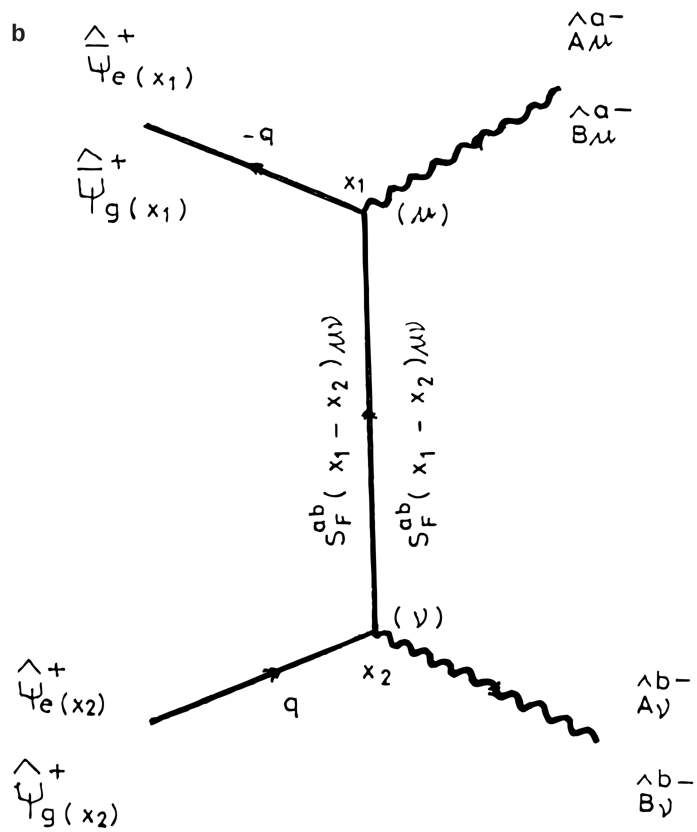

Fig. 4. Continued.

operators of the form $\hat{\Psi}_e^+$, $\hat{\bar{\Psi}}_e^+$, $\hat{\Psi}_e^-$, $\hat{\Psi}_e^-$, $\hat{\Psi}_g^+$, $\hat{\bar{\Psi}}_g^+$, $\hat{\bar{\Psi}}_g^-$, and $\hat{\Psi}_g^-$ which absorb and create the particles present initially and finally. Equation (3.40) can be written as

$$\hat{S}_{3a}^{(2)} = -\frac{ie^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_e^- \hat{\gamma}_\mu T^a \hat{\Psi}_e^+)_{x_1} (\hat{\bar{\Psi}}_e^+ \hat{\gamma}_\upsilon T^b \hat{\Psi}_e^-)_{x_2}]\{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon}$$
$$- \frac{ig^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g^- \hat{\gamma}_\mu T^a \hat{\Psi}_g^+)_{x_1} (\hat{\bar{\Psi}}_g^+ \hat{\gamma}_\upsilon T^b \hat{\Psi}_g^-)_{x_2}]\{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon} \tag{3.41a}$$

$$\underset{x_1 \leftrightarrow x_2}{\overset{\mu \leftrightarrow \upsilon}{a \leftrightarrow b}} = -\frac{ie^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_e^+ \hat{\gamma}_\mu T^a \hat{\Psi}_e^-)_{x_1} (\hat{\bar{\Psi}}_e^- \hat{\gamma}_\upsilon T^b \hat{\Psi}_e^+)_{x_2}]\{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon}$$
$$- \frac{ig^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g^+ \hat{\gamma}_\mu T^a \hat{\Psi}_g^-)_{x_1} (\hat{\Psi}_g^- \gamma_\upsilon T^b \hat{\Psi}_g^+)_{x_2}]\{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon}$$
$$= \hat{S}_{3b}^{(2)} \tag{3.41b}$$

a

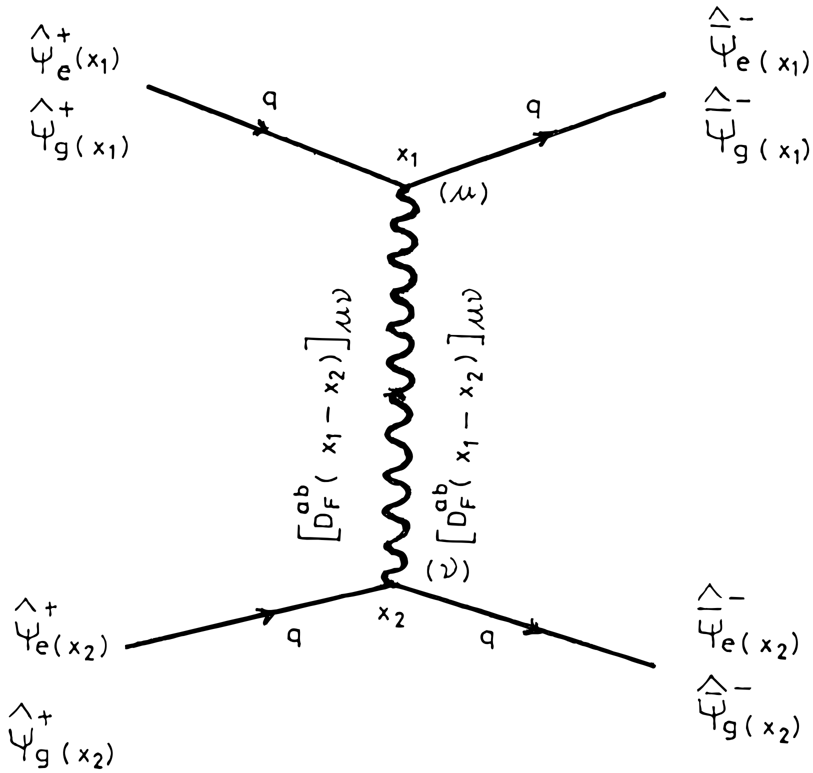

Fig. 5. (a) Dyon–dyon scattering in non-Abelian gauge theory. (b) Dyon–antidyon scattering in non-Abelian gauge theory.

$$\begin{aligned}\hat{S}^{(2)}_{3c} = &-\frac{ie^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^-_e \hat{\gamma}_\mu T^a \hat{\Psi}^-_e)_{x_1} (\hat{\bar{\Psi}}^+_e \hat{\gamma}_\upsilon T^b \hat{\Psi}^+_e)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\upsilon} \\ &-\frac{ig^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^-_g \hat{\gamma}_\mu T^a \hat{\Psi}^-_g)_{x_1} (\hat{\bar{\Psi}}^+_g \hat{\gamma}_\upsilon T^b \hat{\Psi}^+_g)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\upsilon}\end{aligned} \tag{3.42a}$$

$$\begin{aligned}a \overset{\mu\leftrightarrow\upsilon}{\underset{x_1\leftrightarrow x_2}{\leftrightarrow}} b = &-\frac{ie^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^+_e \hat{\gamma}_\mu T^a \hat{\Psi}^+_e)_{x_1} (\hat{\bar{\Psi}}^-_e \hat{\gamma}_\upsilon T^b \hat{\Psi}^-_e)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\upsilon} \\ &-\frac{ig^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}^+_g \hat{\gamma}_\mu T^a \hat{\Psi}^+_g)_{x_1} (\hat{\Psi}^-_g \gamma_\upsilon T^b \hat{\Psi}^-_g)_{x_2}]\{D^{ab}_F(x_1 - x_2)\}_{\mu\upsilon} \\ = &\ \hat{S}^{(2)}_{3d}\end{aligned} \tag{3.42b}$$

where the terms $\hat{S}^{(2)}_{3a}$ and $\hat{S}^{(2)}_{3c}$ are shown in Figs. 5a and 5b, respectively.

b

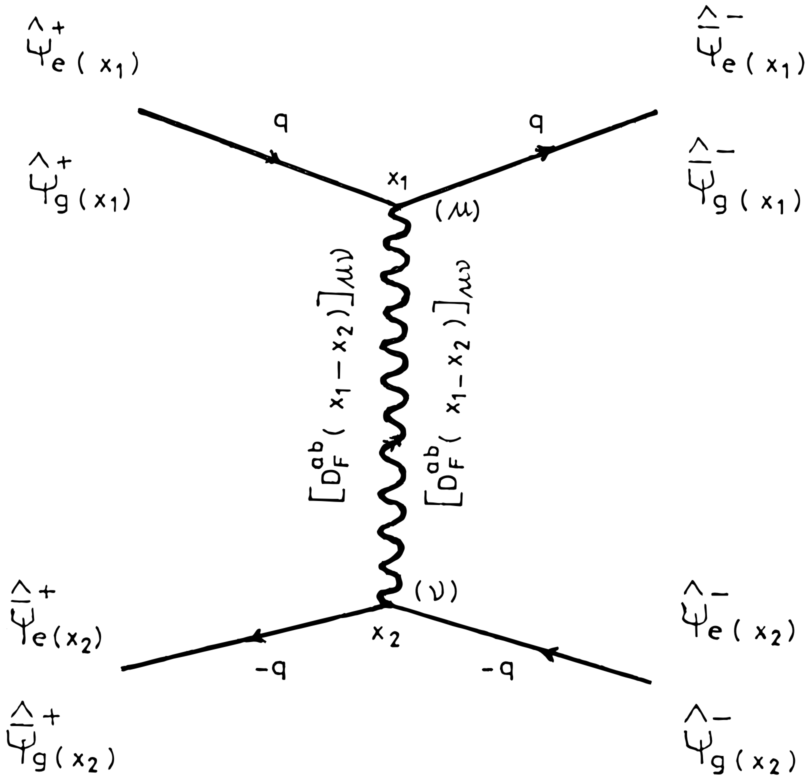

Fig. 5. Continued.

We can write $\hat{S}_4^{(2)}$ in the following way:

$$\hat{S}_4^{(2)} = -e^2 \int d^4x_1 \int d^4x_2\, N[(\bar{\hat{\Psi}}_e \hat{\bar{\gamma}}_\mu \hat{\bar{A}}^a_\mu T^a \hat{\Psi}_e)_{x_1} (\bar{\hat{\Psi}}_e \hat{\bar{\gamma}}_\upsilon T^b \hat{\bar{A}}^b_\upsilon \hat{\Psi}_e)_{x_2}]$$

$$- g^2 \int d^4x_1 \int d^4x_2\, N[(\bar{\hat{\Psi}}_g \hat{\bar{\gamma}}_\mu \hat{\bar{B}}^a_\mu T^a \hat{\Psi}_g)_{x_1} (\bar{\hat{\Psi}}_g \hat{\bar{\gamma}}_\upsilon T^b \hat{\bar{B}}^b_\upsilon \hat{\Psi}_g)_{x_2}] \tag{3.43}$$

This contains four uncontracted fermion field operators leading to two processes depending on whether the initial and final fermion is a non-Abelian dyon or an antidyon. For the non-Abelian dyon case Eq. (3.43) reduces to

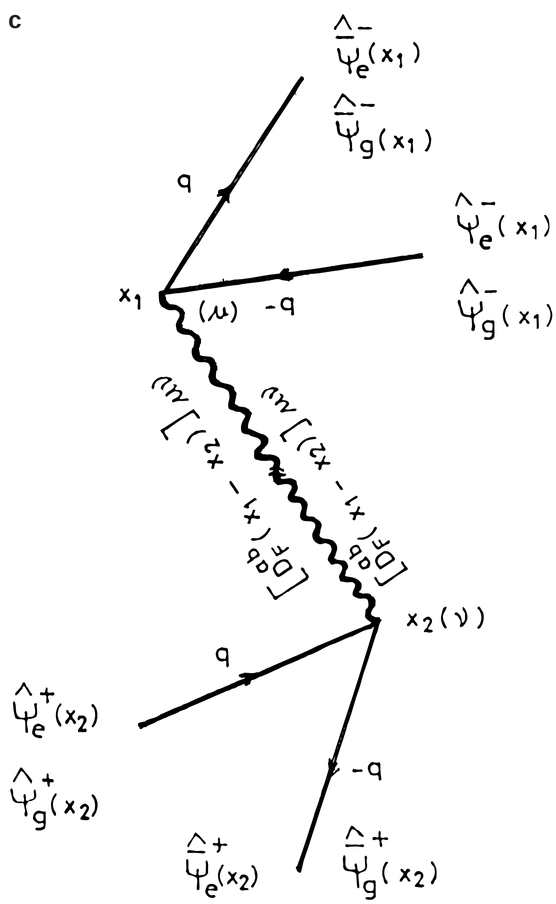

Fig. 5. Continued.

$$
\begin{aligned}
\hat{\bar{S}}_{4q}^{(2)} = & -e^2 \int d^4x_1 \int d^4x_2 \, \hat{\bar{\Psi}}_e^-(x_1) \hat{\bar{\gamma}}_\mu T^a S_F^{ab}(x_1 - x_2)_{\mu\upsilon} \hat{\bar{\gamma}}_\upsilon T^b \hat{\Psi}_e^+(x_2) \{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon} \\
& - g^2 \int d^4x_1 \int d^4x_2 \, \hat{\bar{\Psi}}_g^-(x_1) \hat{\bar{\gamma}}_\mu T^a S_F^{ab}(x_1 - x_2)_{\mu\upsilon} \hat{\bar{\gamma}}_\upsilon T^b \hat{\Psi}_g^+(x_2) \{D_F^{ab}(x_1 - x_2)\}_{\mu\upsilon}
\end{aligned}
\tag{3.44}
$$

which represents the self-energy of the non-Abelian dyon as shown in Fig. 6.

The term $\hat{\bar{S}}_5^{(2)}$ leads to photon self-energy in non-Abelian gauge theory. For this process $\hat{\bar{S}}_5^{(2)}$ can be written as

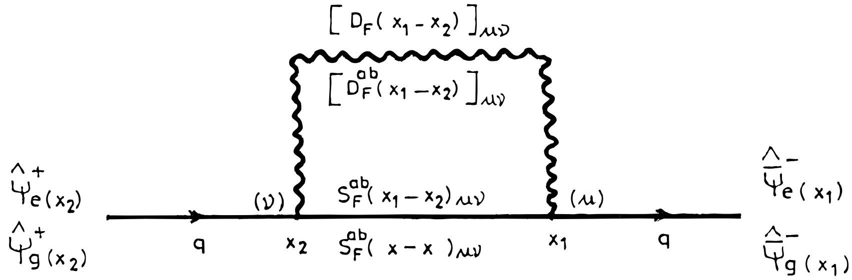

Fig. 6. Self-energy of a non-Abelian dyon.

$$\hat{S}_5^{(2)} = -\frac{e^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_e\hat{\gamma}_\mu \hat{A}_\mu^a T^a \hat{\Psi}_e)_{x_1}(\hat{\bar{\Psi}}_e\hat{\gamma}_\nu \hat{A}_\nu^b T^b \hat{\Psi}_e)_{x_2}]$$

$$-\frac{g^2}{2!}\int d^4x_1 \int d^4x_2\, N[(\hat{\bar{\Psi}}_g\hat{\gamma}_\mu \hat{B}_\mu^a T^a \hat{\Psi}_g)_{x_1}(\hat{\bar{\Psi}}_e\hat{\gamma}_\nu \hat{B}_\nu^b T^b \hat{\Psi}_g)_{x_2}] \tag{3.45}$$

which is shown in Fig. 7. The term $\hat{S}_6^{(2)}$ does not have any uncontracted part and the corresponding Feynnman diagram, known as the vacuum diagram, is shown in Fig. 8.

4. CONCLUSION

Equation (2.4) describes the Compton shift in non-Abelian dyon–photon scattering. This equation shows that we need two photons for obtaining a Compton shift of 0.02428571 Å in non-Abelian dyon–photon scattering instead of the usual one responsible for electron–photon scattering of quantum electrodynamics. One of these two photons is responsible for electron–photon scattering and the other is responsible for monopole–photon scattering. This scattering process is similar to that in Abelian dyon–photon scattering.[19,20] The energy of the photon associated with magnetic charge is enormously high (2.42906×10^{14} GeV), which is much greater than the energy of the photon responsible for the magnetic charge of a dyon in Abelian gauge theory (9.332885×10^{-12} J). Equation (3.6) is the Lagrangan density for a non-Abelian dyon moving in the generalized electromagnetic field of another

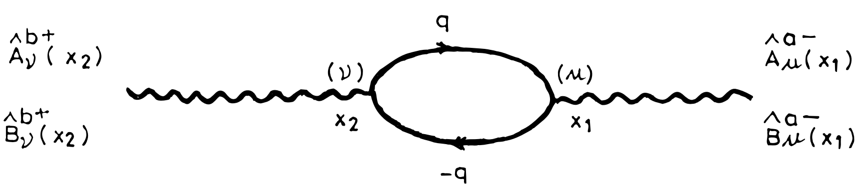

Fig. 7. Photon self-energy in non-Abelian gauge theory.

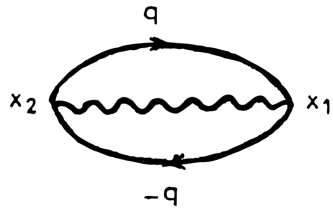

Fig. 8. Vacuum diagram.

non-Abelian dyon. Equation (3.14) is the *S*-matrix expansion and with the help of Eq. (3.15) we have obtained different scattering processes described by Eqs. (3.28), (3.30), (3.33), (3.38), (3.40), and (3.41a)–(3.45) and shown in Figs. 2–8. As such the photons associated with dyons in Abelian as well as in non-Abelian gauge theory are different from the photon associated with the electron, in agreement with results of others,[23–28] where it has been conjectured that the exact gauge group for the monopole system is not $SU(3) \times SU(2) \times U(1)$, but could be $SU(3) \times SU(2) \times U(1) \times U'(1)$. Moreover, occurence of monopoles and dyons in heterotic string theory[29] confirms that very high energy is associated with these particles.

The energy of photons responsible for dyon–photon scattering in non-Abelian gauge theory is much higher than the energy of photons responsible for dyon–photon scattering in Abelian gauge theory,[19,20] which clearly shows that massive fields in terms of the Higgs triplet play a major role in describing the scattering processes.

REFERENCES

1. G.'t Hooft (1974), *Nucl. Phys. B* **79**, 276.
2. A. M. Polyakov (1974), *JETP Lett.* **20**, 194.
3. B. Julia and A. Zee (1975), *Phys. Rev. D* **11**, 2227.
4. M. K. Prasad and C. M. Sommerfield (1975), *Phys. Rev. Lett.* **35**, 760.
5. C. P. Dokos and T. N. Tomaros (1980), *Phys. Rev. D* **21**, 2940.
6. J. P. Preskill (1984), *Annu. Rev. Nucl. Part. Sci.* **34**, 461.
7. Y. M. Cho (1980), *Phys. Rev. D* **21**, 1080.
8. G.'t Hooft (1981), *Nucl. Phys. B* **190**, 455; (1979), **153**, 141.
9. V. Rubakov (1981), *JETP Lett.* **33**, 645.
10. V. Rubacov (1982), *Nucl. Phys. B* **203**, 311.
11. E. Witten (1979), *Phys. Lett. B* **86**, 283.
12. B. Cabrera (1982), *Phys. Rev. Lett.* **48**, 1378.
13. B. S. Rajput and D. C. Joshi (1981), *Hadronic J.* **4**, 1805.
14. B. S. Rajput and D. S. Bhakuni (1982), *Lett. Nuovo Cimento* **34**, 509.
15. B. S. Rajput, S. R. Kumar; and O. P. S. Negi (1983), *Indian J. Pure Appl. Phys.* **21**, 638.
16. B. S. Rajput and S. R. Kumar (1983), *Indian J. Pure Appl. Phys.* **21**, 25.
17. V. P. Pandey and B. S. Rajput (1996), *Nuovo Cimento B* **111**, 275.
18. V. P. Pandey and B. S. Rajput (1999), *Prog. Theor. Phys.* **101**, 1165.

19. P. P. Purohit, V. P. Pandey, and B. S. Rajput (1999), *Indian J. Pure Appl. Phys.* **37**, 163.
20. P. P. Purohit, V. P. Pandey, and B. S. Rajput, *Can. J. Phys.*, submitted.
21. P. P. Purohit, V. P. Pandey, and B. S. Rajput, *Indian J. Pure Appl. Phys.* (in press).
22. J. P. Preskill (1979), *Phys. Rev. Lett.* **43**, 1365; A. H. Guth (1981), *Phys. Rev.* **23**, 347; A. H. Guth and S. H. Tye (1980), *Phys. Rev. Lett.* **44**, 631; T. P. Cheng and C. F. Li (2000), *Gauge Theory of Elementary Particles*, Clarenden Press, Oxford, p. 473.
23. D. I. Olive (1976), *Nucl. Phys. B* **113**, 413; E. Comgan and D. I. Oliver (1976), *Nucl. Phys. B* **110**, 237.
24. P. Goddard, J. Nuyts, and D. Olive (1977), *Nucl. Phys. B* **125**, 1.
25. V. A. Rubakov (1983), *Phys. Lett. B.* **120**, 191.
26. S. M. Barr, D. B. Reiss, and A. Zee (1983), *Phys. Rev. Lett.* **50**, 317.
27. J. M. S. Rana, H. C. Chandola, and B. S. Rajput (1989), *Prog. Theor. Phys.* **82**, 153.
28. N. Cabibbo and E. Ferrari (1962), *Nuovo Cimento* **23**, 1147.
29. Ashok Sen (1993), *Phys. Lett. B* **303**, 22.